Elementary students' conceptions of steepness

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Algebra and linearity

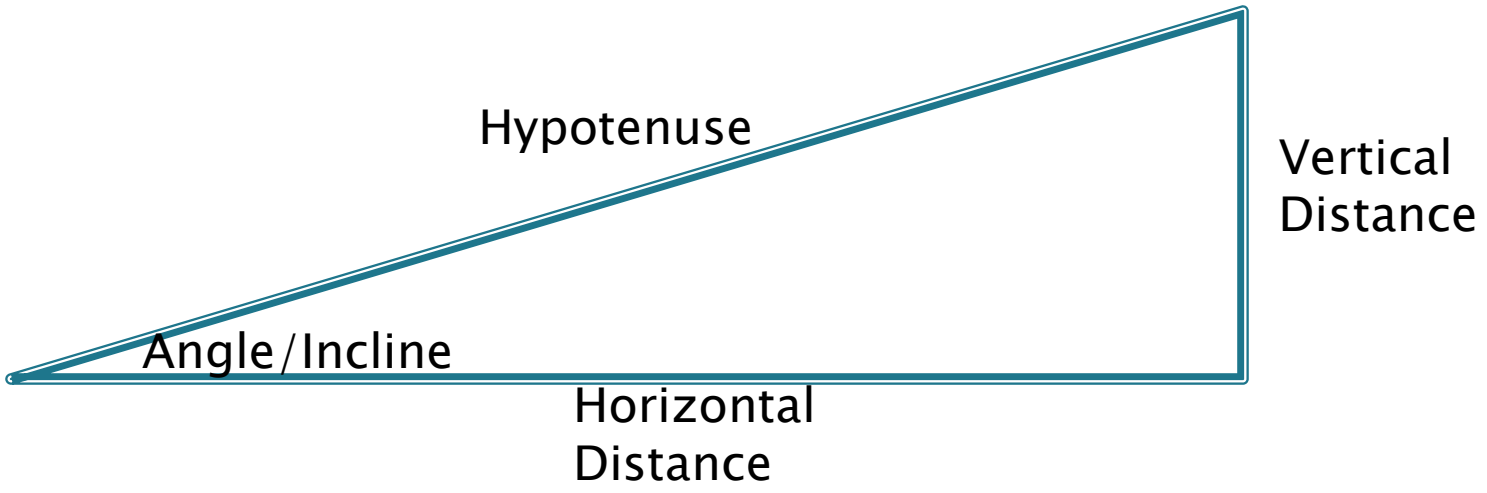
- Importance of Algebra
  - NCLB
  - NCTM
  - State Frameworks
- Linearity in Algebra
- Slope
  - central concept in linearity

Slope

- Central concept in linearity
  - Ratio of *vertical* distance and the *horizontal* distance
  - "rise over run"
  - $\frac{y_2 - y_1}{x_2 - x_1}$
  - $\frac{\Delta y}{\Delta x}$

Research Question – Pilot Study

- Which dimensions do students attend to and neglect when describing steepness?



Cheng, D. S., & Sabinin, P. D. (2007). *Elementary students' conceptions of steepness:* Boston University.

Students we interviewed

- Very small sample for pilot study
  - 8 students
    - Grades 2 – 7 (except 6)
    - 3 boys, 5 girls
  - 6 from high, 2 from low-scoring school districts

Concrete – Scenarios #1&2

Concrete – Scenarios #4&5

Coding Structure

- "Which ramp is steeper?"
  - *"Task Accuracy"*
    - Correct
    - Incorrect
- "How do you know?"
  - *"Explanation Accuracy"*
    - Correct
    - Incorrect
  - *"Mathematical Categories"*
    - Vertical
    - Horizontal
    - Hypotenuse
    - Incline
    - Combination
    - Area
    - Speed
    - Other

Results across the grades

(Note: only 8 students)

- No increase in number of explanations
  - Overall (R^2=0.061)
  - Per task (R^2=0.089)
- No increase in accuracy
  - Task (R^2=0.063, Range 71% – 88%)
  - Explanation (R^2=0.041, Range 45% – 90%)
- Most frequently used dimensions
  - Vertical (40%)
  - Hypotenuse (11%)
  - Incline (14%)

Initial Implications for Elementary Classrooms

- Students know a lot about steepness
  - Correct and incorrect prior knowledge
- Most of this knowledge is intuitive, not formalized
- No increase in formalization across grades*
- We should begin to help students reason about steepness as early as possible

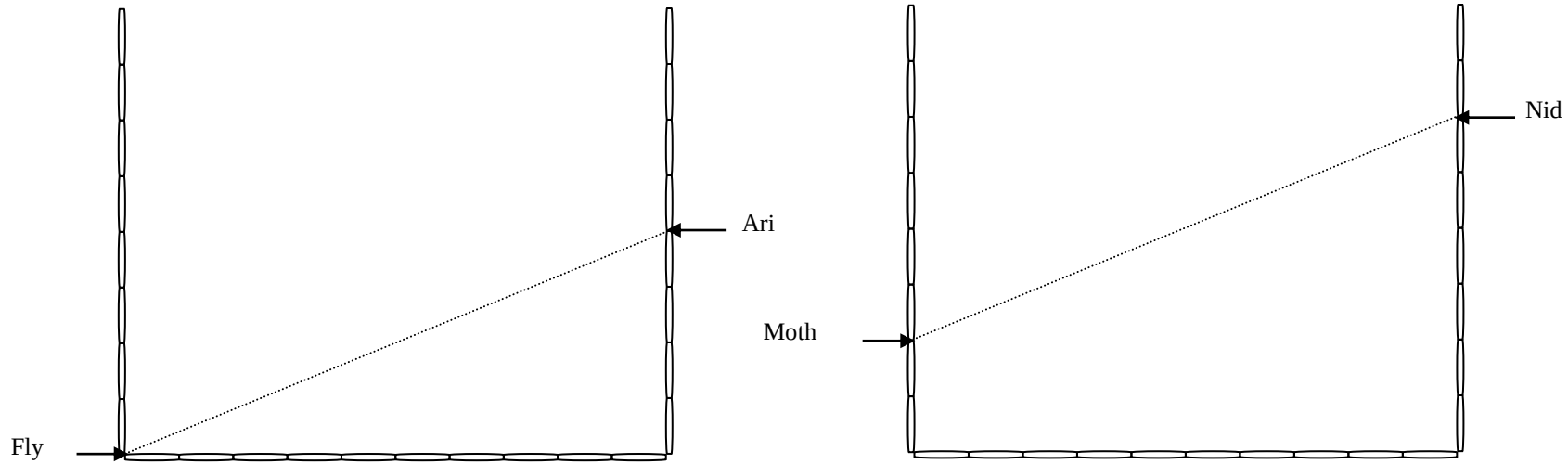
*Note: only 8 students

Steepness Questionnaire 'Ari & Nid'

- Equal emphasis on vertical and horizontal
- De-emphasizes hypotenuse
- Structured based on development of cognitive complexity
- Reliability – currently obtaining Test/Retest results
- Validity – how do we show what it is measuring?
- Uses
  - Paper-and-pencil assessment
  - Interview activities
  - Small group activities
  - In-class activities

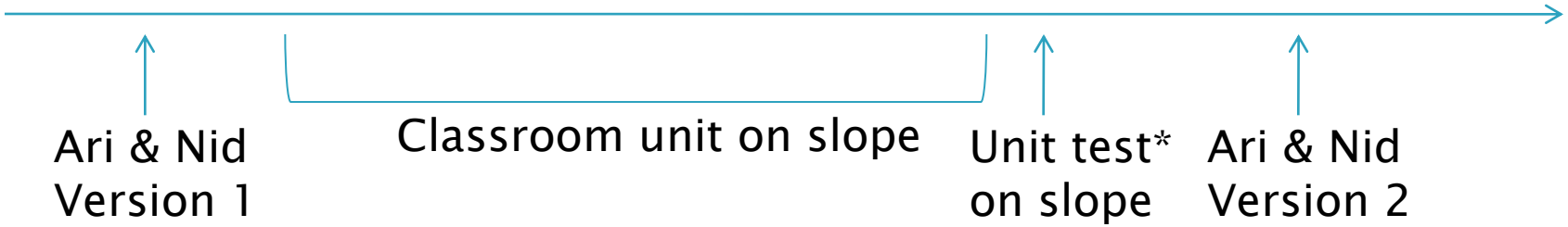
Ari & Nid's World

- Ari & Nid are spiders living in a tiled room
- They shoot straight webs of infinite length
- Steeper webs are more accurate



Ari & Nid – Part 1

- *Does a student's understanding of steepness correlate with his or her success in the formal learning of slope in Algebra courses?*
- Design



* Containing 3 – 4 standardized questions on slope that will be the same across all groups.

- Analysis
  - Correctness of answers
  - Types of explanations
  - Correlation between the Ari & Nid and the standardized slope questions

Ari & Nid – Part 2

- *How pervasive is the misconception that the area under a ramp varies directly with its steepness in upper elementary and middle school?*
- Design
  - Ari & Nid as pencil-and-paper test
  - Grades 5, 7, and 9
  - Five Boston-area districts
    - Curriculum supervisors will select 'average' classes
    - Two classes per grade per district
- Analysis
  - Association between grade level and correctness
    - Overall and the 'Area under the ramp' misconception
  - Percent of students who hold the 'Area under the ramp' misconception

Ari & Nid – Part 3

- *How persistent is the above misconception as measured by the amount of cognitive conflict required for the students to change their mind?*
- Design
  - 15 – 20 one-on-one interviews with students who have the 'Area under the ramp' misconception
    - Questions will provide an increasing level of cognitive conflict with the misconception
- Analysis
  - The level of cognitive conflict at which the student abandons misconception
  - Student's explanations for solutions of each question

Potential Impact

- Informing the design of curricular slope-readiness supplements for elementary schools
- Informing the design of professional development for elementary school teachers
- Expand the understanding of students' development of the concept of steepness