MACS Bridgewater State College

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Hands on Geometry

Grades 7 & 8

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Agenda

1. Paper Folding – icosahedron

2. Patty Paper
    a. Make a protractor
    b. And a lot more

3. Inductive and Deductive Reasoning

4. Vesica Piscis

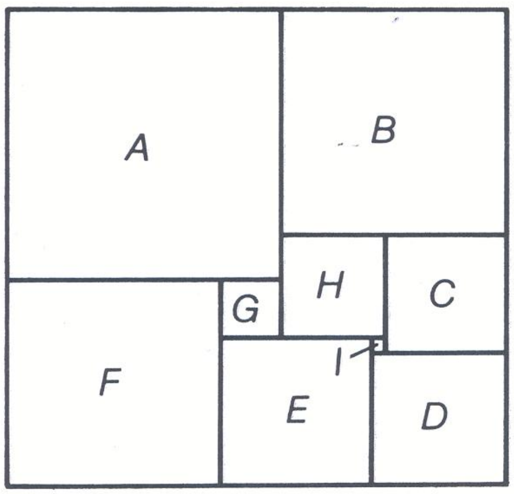
5. Area of Triangle

6. Contest Problems for those who'd like the challenge…

7. The Shadow Knows

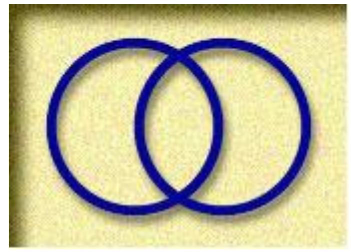
SQUARE UNITS

The following diagram shows an arrangement of squares that appeared on the cover of the November 1958 issue of *Scientific American*. If the area of square *C* is 64 square units and the area of square *D* is 81 square units, what are the areas of the other seven squares?



What are the dimensions of the big shape?

Vesica Piscis



The Vesica Piscis is a symbol made from two circles of the same radius, intersecting in such a way that the center of each circle lies on the circumference of the other circle. The term "vesica piscis" is first recorded in literature in 1809, but is no doubt much older. The Vesica Piscis is made by linking two cicles together, bringing the outside edge of each to the midway point of the other. The almond-shaped center of the image is called a mandorla (Latin for almond; "vesica" or "vesica piscis" is sometimes also used to describe only this almond-shaped center.) The mandorla can easily be seen as a grail or chalice, connecting the symbol to Avalon.

(http://www.philomuse.com/kingfisher/lab/vp.htm)

Construct Vesica Piscis:

Draw circle with radius 2in and center A. Make a point B on the circle to the right of point A. Construct a circle with radius 2in and center at point B. Label the intersection points of the circles C and D (top to bottom respectively).

1. Draw segments AB, AC, and CB. What kind of a triangle do you think this is? Prove your observation.

2. What can you tell about quadrilateral ACBD. Prove your hypothesis.

3. Draw segment CD. Label the intersection point between segments AB and CD as point X. What is the measure of ∠ ACX?

4. Continue segment AB so that it intersects circle A at point P. Find the measures of the interior angles of triangle PAC.

5. Draw a line parallel to CD through point B. Label the intersection points of the line with circle B as points M and N. Find the measure of ∠ MBP.

6. Find the measures of the following segments:

a) AC = ?

b) XM = ?

c) AM = ?

d) PC = ?

e) PM = ?

Playing with Patty Paper

Remember to label everything you make with a pencil!

Make line L_1.

Make line L_2 that intersects L_1.

What do you notice?

Make line L_3 that is perpendicular to L_1.

Make line L_4 that is parallel to L_1.

How did you make L_4? What do you notice?

Take a new piece of Patty Paper.

Make an acute angle using two lines. Label the angle with the letter a.

What do you know about complementary angles?

Make an angle *b* which is complementary to a.

What do you know about supplementary angles?

Make an angle *c* which is supplementary to *a*.

<u>Take a new piece of Patty Paper.</u>

Draw two parallel lines L_1 and L_2. Make another line L_3 that intersects both of these lines. Labe all the angles with letters *a - h*.

What do you notice? Write three of your conjectures here and prove them with patty paper. *[Hint: You may choose to use more than one piece of patty paper to help with the proofs.]*

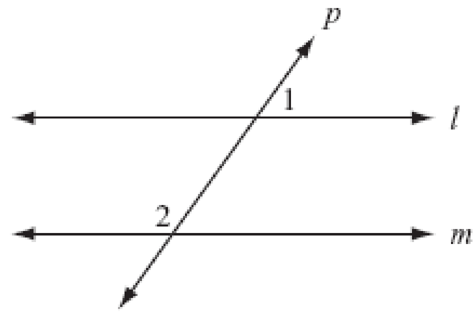
Adapted from Serra, M. 1994. Patty Paper Geometry. Key Curriculum Press

Patty Paper vs. MCAS

2007 MCAS - Grade 8   Standard: 8.G.3

In the figure to the right, parallel lines l and m are intersected by transversal p.

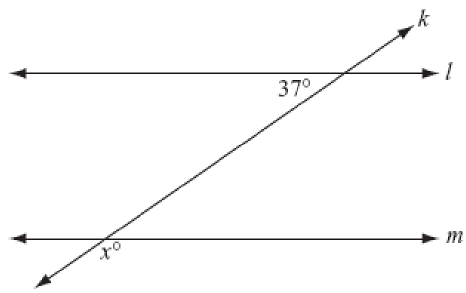
If the measure of ∠1 is 50°, what is the degree measure of ∠2?



2008 MCAS - Grade 10   Standard: 10.G.3

In the diagram to the right, line l is parallel to line m, and line k intersects both lines.

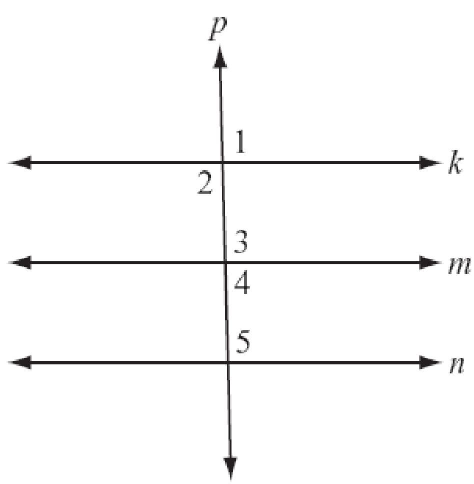
Based on the angle measure in the diagram, what is the value of x?



2009 MCAS - Grade 10   Standard: 10.G.3

In the diagram to the right, lines k, m, and n are parallel lines intersected by line p.
Line p is not perpendicular to lines k, m, and n.

Which of the following angles has a measure that is not equal to the measure of ∠1?



A little bit of Algebra with Patty Paper

Take a Patty Paper and make a fold parallel to one of the sides (try not to make the fold in the middle). You have split one of the sides into two lengths: label one of the lengths *M*, and the other *N*.

Make another fold (perpendicular to the first) to get two squares and two rectangles. Label all the lengths that you see.

What do you notice?

Can you write the area of the patty paper in terms of M and N?

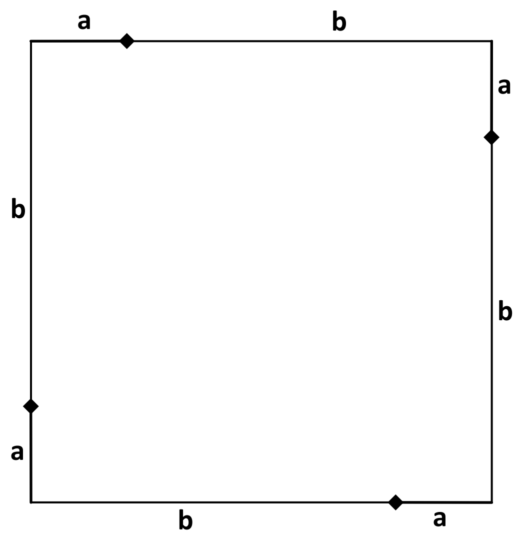
Can you write the area in another way, but still in terms of M and N?

What rule have you just proved?

Adapted from Serra, M. 1994. Patty Paper Geometry. Key Curriculum Press

<u>Another Theorem with Patty Paper - I</u>

Take a piece of Patty Paper and make a mark on each of the sides as you see in the diagram below. Label the lengths *a* and *b*.



Fold in each of the corners to the marks to make four congruent triangles and label their hypotenuses as *c*.

Can you write the area of the patty paper in terms of *a*, *b* and *c*?

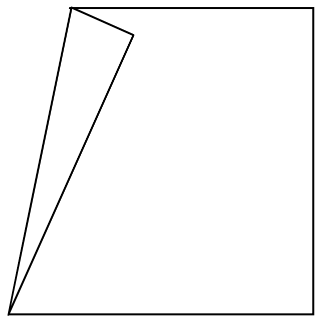
Can you write the area in another way, but still in terms of *a*, *b* and *c*?

What rule have you just proved?

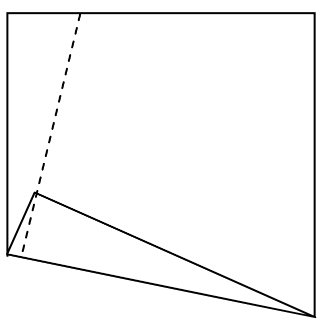
Adapted from Serra, M. 1994. Patty Paper Geometry. Key Curriculum Press

Another Theorem with Patty Paper - II

Take a piece of Patty Paper. Make a fold on one corner to make a triangle as shown below.



Make a fold perpendicular to the first fold and going through an adjacent vertex of the square.



Continue with the other two corners. Then, locate 4 congruent right triangles and a square. Label the hypotenuse of the triangles as *c*, and the two legs as *a* and *b*.

What is the length of the sides of the square?

Can you write the area of the patty paper in terms of *a*, *b* and *c*?

Can you write the area in another way, but still in terms of *a*, *b* and *c*?

What rule have you just proved?

Adapted from Serra, M. 1994. Patty Paper Geometry. Key Curriculum Press

Exploring Patty Paper with Pythagorean Theorem

Show 1 unit, 2 units, $\sqrt{2}$ units, $\sqrt{5}$ units, $2\sqrt{2}$ units on your piece of Patty Paper.

What other lengths can you make using this patty paper?

Construct a square with 1/4 the area of the patty paper.

What are its dimensions?

Construct a square with 1/2 the area of the patty paper.

What are its dimensions?

What other squares can you construct?

Interior Angles in Triangles

Take a new piece of Patty Paper.

Make 3 lines that make a triangle. Try not to make your triangle very small.

Label the interior angles of you triangle with an arc and name them with letters a, b, and c.

What do you notice? Write three of your conjectures here and prove them with your patty paper.

What do you know about the sum of the interior angles (a + b + c) of your triangle?

Prove you conjecture with the patty paper. Can you prove it in another way?

Interior Angles in Other Polygons

Now that we know lots of stuff about triangles, let's find some information about other figures.

Use folds to construct a quadrilateral on your patty paper. Label all its interior angles with a single arc (∢).

Use folds to break it into triangles.

How many triangles did you make?

What can you say about the sum of the interior angles of a quadrilateral?

Explore a pentagon, hexagon, and a heptagon in the same way. Fill in the table below.

Figure	Number of vertices	Number of triangles	Sum of interior angles
Triangle			
Quadrilateral			
Pentagon			
Hexagon			
Heptagon			
n-gon			

Now, look at the formula and explain why it makes sense based on how you derived it.

Adapted from Serra, M. 1994. Patty Paper Geometry. Key Curriculum Press

Interior and Exterior Angles in Polygons

Take a new piece of Patty Paper.

Make 3 lines that make a triangle. Try not to make your triangle very small.

Label the interior angles of you triangle with a single arc.

For each interior angle, label an exterior angle with a double arc.

What do you notice? Write two of your conjectures here and prove them with your patty paper.

What is the sum of exterior angles of a triangle? How do you know?

What can you tell about the sum of the interior and exterior angles that you labeled?

Final thoughts: Do you notice anything else?

<u>Take a new piece of Patty Paper.</u>

Use folds to construct a quadrilateral on your patty paper. Try not to make your quadrilateral very small.

Label the interior angles of you quadrilateral with an arc. For each interior angle, label an exterior angle with a double arc.

What is the sum of exterior angles of a quadrilateral? How do you know?

What can you tell about the sum of the interior and exterior angles that you labeled?

What is the sum of all interior angles of the quadrilateral?

Explore a pentagon, hexagon, and a heptagon in the same way. Fill in the table below.

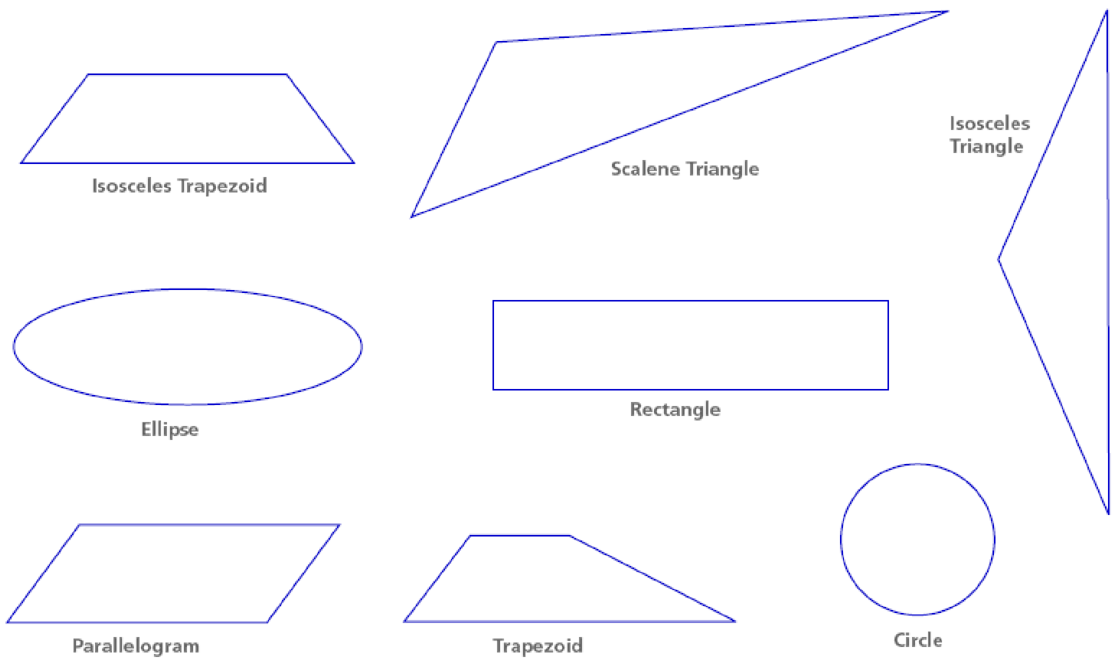
Figure	Total sum of interior and exterior angles	Sum of exterior angles	Sum of interior angles
Triangle			
Quadrilateral			
Pentagon			
Hexagon			
Heptagon			
n-gon			

Now, look at the formula and explain why it makes sense based on how you derived it.

Adapted from Serra, M. 1994. Patty Paper Geometry. Key Curriculum Press

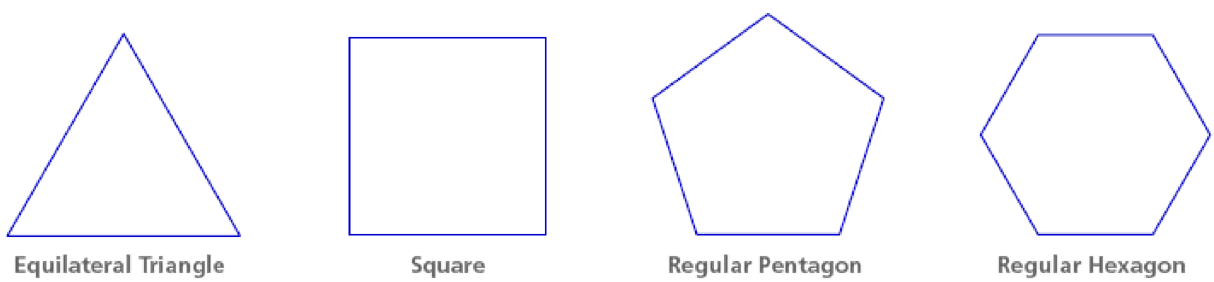
<u>Symmetry</u>

Use patty paper to find all the lines of symmetry you can for the figures below. You can start by tracing the figure on a sheet of patty paper.



Use patty paper to determine if the figures above have rotational symmetry. Determine the angle of rotation where possible.

Use patty paper to find all the lines of symmetry for the regular polygons below. Generalize a rule about the number of lines of symmetry for regular polygons.

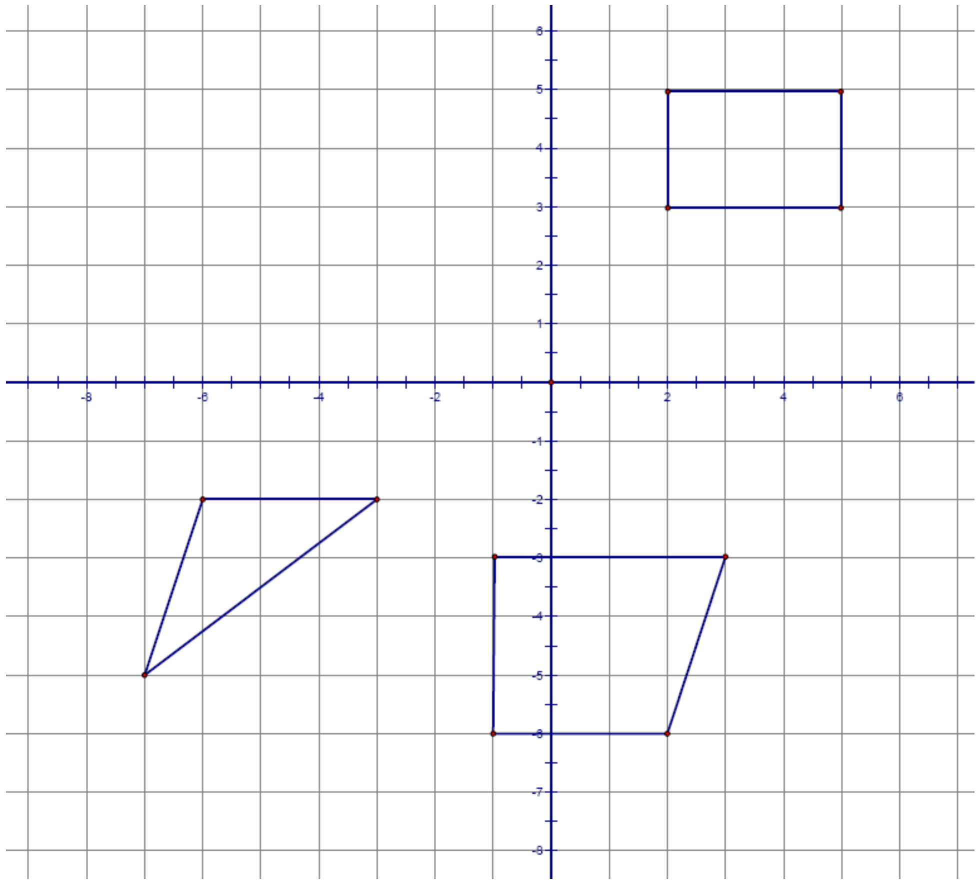


Use patty paper to draw figures with 1, 2, and 4 lines of symmetry. Do any of these figures have rotational symmetry? If so, find the angle of rotation.

Source Unknown

Transformations on Coordinate Plane

Use patty paper to translate each of the figures drawn by 2 units to the left and 1 unit up. Start by carefully tracing the figures onto the patty paper.

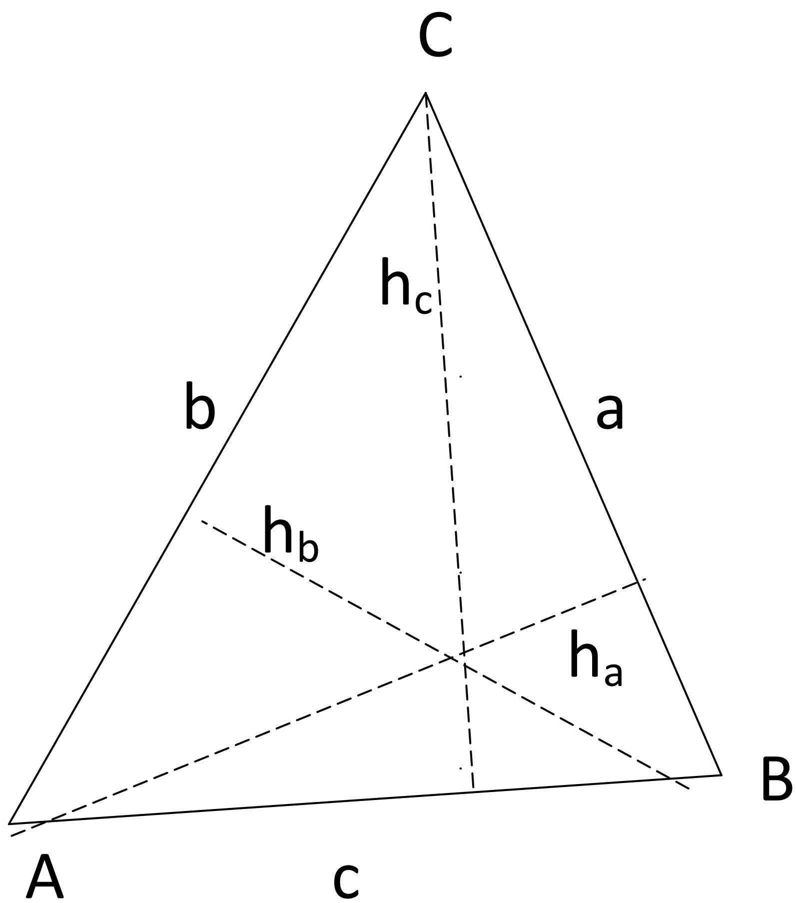


Complete the table below:

Figure	Original Coordinates	Coordinates after translation by 2 units to the left and 1 unit up
Triangle		
Trapezoid		
Rectangle		
Parallelogram	(12,1) (17,1) (11,-2) (16,-2)	
Square		(-6,5) (-3,5) (-6,2) (-3,2)

What can you conclude from this exercise?

Area of a Triangle

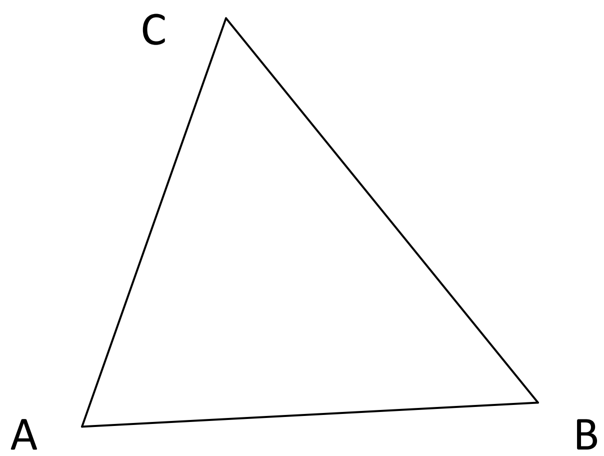


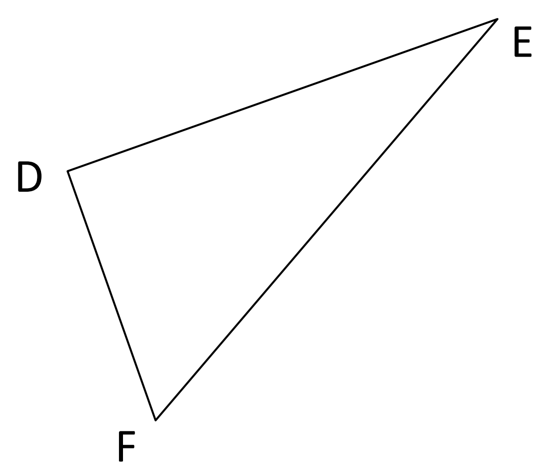
$A_{\blacktriangle ABC} = \frac{1}{2}\, a * h_a$

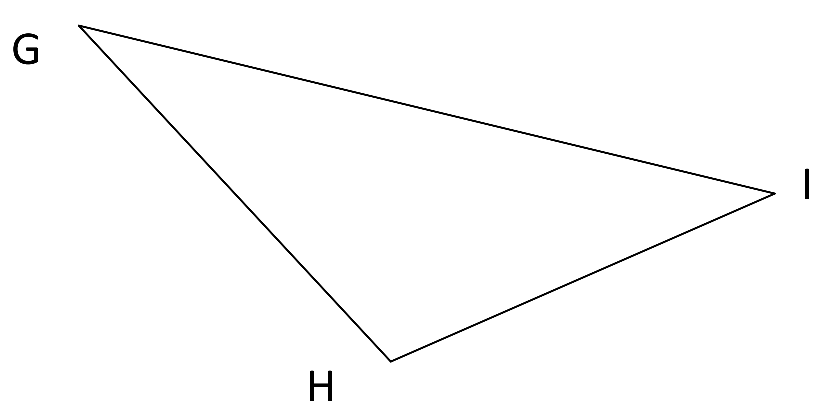
$A_{\blacktriangle ABC} = \frac{1}{2}\, b * h_b$

$A_{\blacktriangle ABC} = \frac{1}{2}\, c * h_c$

Construct all the altitudes on each triangle, label them, and write the formula for the area of each triangle in three different ways.







Solve without using the Pythagorean theorem:

The three sides of a right triangle are 6cm, 8cm, and 10cm. Find the length of the altitude to the hypotenuse.

Challenge for the Math Soul:

Points M, N, and P are midpoints of sides AB, BC, and CA respectively. If the area of triangle ABC is 42 sq.cm, find the area of:

a) triangle ABP
b) triangle AMP
c) quadrilateral PMNC

The Shadow Knows

John Henry was camping in a large, sparsely wooded area in southern Texas. One day he went for a walk. He packed some food and left at 8:00 a.m. on a cloudless, hot day. He walked for 2 miles with his shadow on his left. Then he walked for 5 miles with his shadow in front of him. Then he walked 3 miles with his shadow on his right. Then he walked 1 mile with his shadow behind him. By this time it was 11:00 a.m. He stopped in a nice meadow and had lunch. Then he fell asleep because he was tired from walking. When he woke up, it was 1:00 p.m. He was a little disoriented from his nap as he set out to walk home. He figured he could reverse his previous distances and shadows and walk back to his camp. So he walked 1 mile with his shadow in front of him. Then he walked 3 miles with his shadow on his left. Then he walked 5 miles with his shadow behind him. Then he walked 2 miles with his shadow on his right. Unfortunately, he didn't arrive back at his camp. Give directions for him to get back to his camp by the shortest route.

From "Crossing the River with Dogs: A Mathematical Adventure"

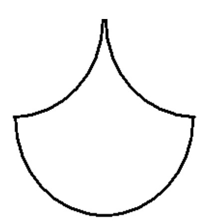
by Johnson and Herr

Source: *Mathematics Contest Problems*

Saint Mary's College

Creative Publications

1. A garden plot is 60 ft. by 40 ft. A walk 2 1/2 ft. wide is to be placed around this plot in such a way that the corner pieces are portions of a circle of radius 2 1/2 ft. with each corner of the plot as center. What is the total area of the walk?
2. A contractor has a large number of pipes 10 in. in diameter. He first makes a row of pipes side by side, and each is in contact with the next on a level surface. Then a second row is placed on this row so as to fit into the hollows between adjacent pipes. He continues this process until he has five rows. What is the total height of the pile of pipes? Give the answer correct to two decimal places.
3. A horse tethered to a rope at one end of a square corral (outside the corral) 10 ft. on a side. The horse can graze at a distance of 18 ft. from the corner of the corral where the rope is tied. What is the total grazing area of the horse?
4. An area is bounded by four quadrant arcs (quarter arcs) of a circle of radius 10 inches, two being in the normal position and two inverted as shown in the figure. What is the area enclosed by the arcs?



Homework:

Student problem:

In a store we can buy squares for 40¢, Circles for 25¢, triangles for 35¢, and rectangles for 45¢. How much would a prism, cube, cylinder, and pyramid cost? List all the possibilities.