

**NCSM 2009
Washington, DC**

**Key components of Effective Professional
Development for Grade 6-8 Teachers:
Pedagogical Content Knowledge, Research
Based, Development of Students'
Mathematical Thinking**

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Institute Title:

<p style="text-align: center;">Understanding Rational Numbers: Fractions, Decimals Percents, and Proportional Reasoning</p>
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Institute Goals:

As a result of the learning experiences in the course:

Participants will become more cognizant of:

- The research and best practices in the area of rational numbers,
- Knowledge package of the concept of rational numbers,
- The meaning of algorithms of operations with fractions, decimals, percents, and proportional reasoning,
- Strategies to teach understanding of rational numbers for students with different readiness levels,
- Looking at student work

Participants will be better able to:

- Vary strategies of performing operations with rational numbers,
- Use variety of models to represent rational numbers,
- Create differentiated problems and lessons,
- Analyze data from students' work,
- Explain the algorithms of operations with rational models,
- Anticipate difficulties with the concept of rational numbers,
- Use students' informal strategies to explain misconceptions

Proposal Components

A. Provided by the PD Provider

1. Goals
2. Key academic concepts
3. Pre/ post assessment items
4. Day-by-day syllabus
5. Technology component
6. Assignments and final project
7. List of required and recommended readings
8. Budget
9. Instructional team structure

B. Provided by the District

10. District's description
11. Rationale for district's needs

Key academic concepts

Content

- Operations with fractions and decimals and solving problems,
- Part-whole and part-set meaning of fractions,
- Solving percent problems,
- Using various models of representation of fractions, decimals, percents to solve problems,
- Ratios and odds.

Pedagogy

- Operations with fractions,
- Meaning of fractions and decimals.

COURSE CONTENT / TOPICAL OUTLINE

June 9, 2008 4:00p.m. – 7:00p.m. (3 hours)

1. *Introduction and Overview.* Participants will get to know each other filling in a “Getting to know you grid.” They will ask each other questions where participants get signatures from other participants about 24 different topics including mathematics related topics.
2. Participants will be provided with the syllabus of the institute. They will discuss the assignments throughout the institute. An opportunity for participants to take the institute for graduate credits will be provided.
3. *Pre-Assessment of Participants*

Reading Assignments:

- Participants will be assigned a chapter from Susan J. Lamon’s book *Teaching Fractions and Ratios for understanding* and will be required to write a one-page summary/reflection of the chapter

July 28, 2008 8:00 – 3:30p.m. (7.5 hours)

Topics of the Day

- Definition of fractions. Exploration of the various meanings of fractions. Understand the concept of fractions as a ratio of whole numbers, as a part of a whole, and as a part of a collection
- Understand and model equivalent fractions
- Solve related problems

Reading Assignments:

- Participants will read Chapter 2 of *Making Sense of Fractions, Ratios, and Proportions*, NCTM.
- Participants will work in groups to answer reflective questions.

July 29, 2008 8:00 – 3:30p.m. (7.5 hours)

Topics of the Day

- Compare, order, and estimate fractions
- Explore the concepts of probability and odds as examples of the meaning of fractions as fractions as part of a whole, part of a set and ratio.
- Strategies for multiplication of fractions
- Problem solving

Reading Assignments:

- Chapter 3, *Knowing and Teaching Elementary Math* by Liping Ma.

July 30, 2008 8:00 – 3:30p.m. (7.5 hours)

Topics of the Day:

- Understand algorithm for division of fractions
- Explore alternative algorithms for division of fractions
- Understand the meaning of percent.
- Use benchmarks to estimate the reasonableness of answers to percent problems.
- Understand and solve percent problems
- Problem solving with fractions and percents

Reading Assignments:

- Participants will choose an article from NCTM journals or other reputable journal and write a review.

August 5, 2008 8:00 – 3:30p.m. (7.5 hours)

Topic of the Day

- Understand, model, and explain addition and subtraction with fractions.

Assignments:

- A detailed outline of the final project will be due at the end of the week. Participants should identify the general topic of their project and begin their outline.

Reading Assignments:

- Participants will read chapter 5 and 7 of *Making Sense of Fractions, Ratios, and Proportions*.

August 6, 2008 8:00 – 3:30p.m. (7.5 hours)

Topics of the Day

- Explore meaning of decimals using models
- Operations with decimal numbers
- Investigate relationships between fractions, decimals, and percents

Assignment:

- A detailed outline of the final project is due on July 10th.

August 7, 2008 8:00 – 3:30p.m. (7.5 hours)

- Investigate relationships between fractions, decimals, and percents (continued)
- Explore proportional reasoning

- Problem solving with proportions
- Participants will share their ideas about final project.
- Post Assessment

Assignment:

- Implementation of final project due November 3rd.

September 20, 2008 8:00 – 3:30 (7.5 hours)

Topic of the Day: Rational Numbers in our world

- Participants will be involved in a scavenger hunt to find rational numbers throughout the mall. Specific assignments, prompts and problems will be given to groups of participants. Findings and solutions will be shared during the second half of the day.

Assignment:

- Implemented projects and samples of student work due on November 8.

November 8, 2008 8:00 – 3:30 (7.5 hours)

Topics of the Day:

- Looking at Student Work

Classroom Implementation

- The instructor will lead participants into reviewing and scoring samples of student work.
- Participants will focus on solving, scoring, and exploring strategies for open-response questions.
- Participants will engage in a discussion about the ongoing implementation of the summer project and about the impact the summer course has had on their classroom teaching.
- Participants will share the implementation of the final projects.
- Participants will be placed in focus groups. They will share their experiences among peers.
- The focus will be on reflections, suggestions, and looking for evidence of learning in student samples.

Assignments:

- A. Daily reflections**
- B. Daily Homework Problem Sets**
- C. Article Review**
- D. Final Project**
 - Create
 - Implement in the fall
 - Collect student samples
 - Analyze results and reflect
 - Present to your colleagues

Assessment Professional Development Institute: Rational Numbers

Problem 1:

If $p = 0.02$, $q = 1.2$, and $r = 0.4$, solve for "?" in the following problems:

1) $? \div r = q$
 $? = \underline{\hspace{2cm}}$

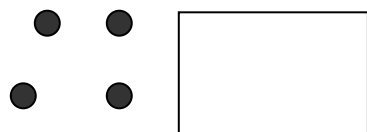
2) $q \div ? = r$
 $? = \underline{\hspace{2cm}}$

3) $q - ? = p$
 $? = \underline{\hspace{2cm}}$

4) $? - r = p$?
 $? = \underline{\hspace{2cm}}$

Problem 2

The rectangle below represents a cover blocking your view of $\frac{5}{7}$ of the circles. Draw the missing circles inside the rectangle.



Explain how you solved the problem.

Problem 3:

If the product of two numbers is 1 and one of the numbers is greater than 1, what do you know about the other number? Explain your answer.

Problem 4:

In an upcoming freshman class of 500 students, only 20 claimed to be math majors.

- a) What percent of the freshman class is this?
- b) When the survey was repeated the next year, 5% of the non-math majors had decided to switch and become math majors.
 - i. How many math majors are there now?
 - ii. What percent of the freshman class do they represent?

Problem 5:

On a street map of London, $\frac{3}{4}$ inch represents 1 mile.

The bowling alley is $\frac{3}{8}$ inch from the supermarket. How many miles is the bowling alley from the supermarket? Explain your reasoning.

Problem 6:

John took all his money out of his bank savings account. He spent \$50 on a radio and $\frac{3}{5}$ of what remained on presents.

Half of what was left he put back in his checking account, and the remaining \$35 he donated to charity. How much money did John originally have in his savings account?

Use a **picture** to solve the problem.

Problem 7:

Show one alternative way to solve $\frac{2}{3} \div \frac{5}{7}$.

Do not use the “invert and multiply method.”

Show your answer in a fraction form.

Problem 8:

There are orange, yellow, and green marbles in a bag.

You pick one marble without looking.

- The odds in favor of picking orange are 3 to 7.
 - The odds in favor of picking yellow are 4 to 6.
 - There are 4 more yellow marbles than green marbles in the bag.
1. How many marbles are in the bag?
Explain.
 2. What is the probability that the chosen marble is green?
Explain.

Groundworks Data and Probability, the Wright Group

Problem 9:

The capacity of an elevator is either 20 children or 15 adults. If two children are currently on the elevator, how many adults can still get on?

From “Crossing the River with Dogs,” Key Curriculum Press

Problem 10:

Orange Juice Problem

For exhibit night at the school some of the students hope to sell lots of orange drink to the thirsty people who attend. Some suggestions for the mixing rule include:

	Number of Cans of	
	Frozen Orange Juice Concentrate	Water
Shantell’s	1	3
Jeremy’s	2	5
Allison’s	3	7
Paolo’s	4	11

Which of these mixes will produce the “most orangy” drink? How do you know?

Connected Mathematics Program – Comparing and Scaling: Ratio, Proportion and Percent. Pearson/Prentice Hall.

Problem 11:

One of the most common errors in adding fractions follows:

$$\frac{2}{3} + \frac{4}{5} = \frac{6}{8}.$$

Choose one of the following manipulatives and describe how you would use them to help to clarify this misconception:

- Number line
- Fraction circles
- Fraction Bars
- Counting chips

Problem 12:

When we multiply a proper fraction by another proper fraction, the product is a fraction that is smaller than either of the factors. Explain why this is true. Include a drawing of a mathematical model in your explanation.

Problem 13:

Many students define a fraction as a number that is less than one whole.

How would you work with a student who holds this misconception?

Include questions, examples, manipulatives, and / or models that would be beneficial.

Problem 14:

Describe a real word situation when you would need to calculate the answer to the following problem: $\frac{5}{6} \div \frac{1}{2}$.

Key Concepts	Problems and Points	Max. points for key concept	Class average pretest score		Class average posttest score		Class average content gain	
			Points	%	Points	%	Points	%
Operations with fractions and decimals and solving problems (content)	Pr. 1 -- 10 pts. Pr. 3 -- 10 pts. Pr. 7 -- 10 pts.	10	5.30	53.00%	6.68	66.83%	1.38	13.83%
Part-whole and part-set meaning of fractions (content)	Pr. 2 -- 10 pts.	10	6.98	69.83%	9.70	97.00%	2.72	27.17%
Solving percent problems (content)	Pr. 4-- 10pts	10	8.90	89.00%	9.05	90.50%	0.15	1.50%
Using various models of representation of fractions, decimals, percents to solve problems (content)	Pr. 5 -- 10 pts. Pr. 6 -- 10 pts. Pr. 10 -- 10 pts.	10	6.98	69.83%	9.38	93.83%	2.40	24.00%
Ratios and Odds (content)	Pr. 8 -- 10 pts. Pr. 9 -- 10 pts.	10	2.83	28.25%	7.73	77.25%	4.90	49.00%
Operations with fractions (pedagogy)	Pr. 11 -- 10 pts. Pr. 12 -- 10 pts. Pr. 14 -- 10 pts.	10	4.78	47.83%	7.05	70.50%	2.27	22.67%
Meaning of fractions and decimals (pedagogy)	Pr. 13 -- 10 pts.	10	7.65	76.50%	7.85	78.50%	0.20	2.00%

A Successful Content Institute:

- Has clear content academic goals
- Focuses on key mathematical ideas
- Has clear expectations for readings, assignments and final product
- Contains a requirement for classroom implementation component
- Has built in opportunities for out of class discussions
- Includes assessment of participants' knowledge
- Has an administrator (e.g. principal, department head) as a participant