

# Observe, Construct, and Imagine: Getting Ready for Algebra

Diana Cheng  
Boston University

Polina Sabinin  
Boston University

November 7, 2008

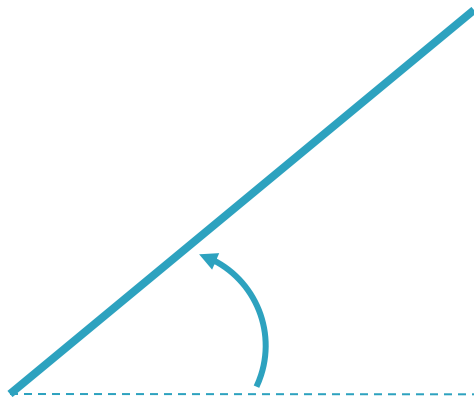


# Early Algebra

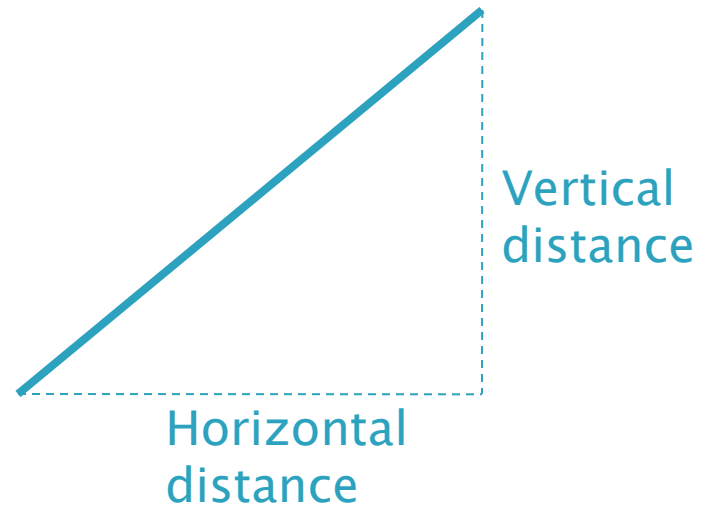
- ▶ Bring algebraic reasoning into elementary grades
- ▶ Examples:
  - Variables in arithmetic activities
  - Geometric and numeric patterns
  - Function machines
  - Rate of change: slope with time as one of the measures
- ▶ We suggest introducing measures of “steepness”
  - Does not appear in most K–5 curricula

# Measures of Steepness

▶ Angular



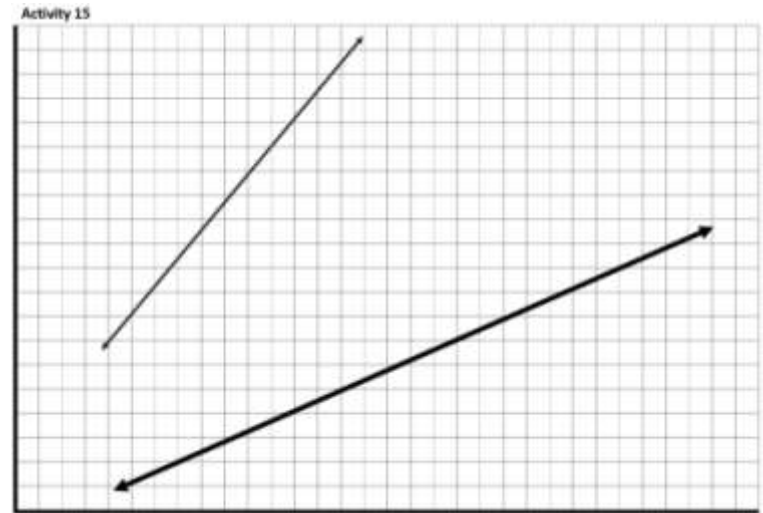
▶ Slope




$$\textit{Slope} = \frac{\textit{Vertical Distance}}{\textit{Horizontal Distance}}$$

# What kids know

- ▶ Angular steepness
  - Physical angle concepts
    - Caution when walking up ramps
    - Choice of locomotion up ramps
  - Abstract angle concepts
    - Static
      - Ramp
      - Bend in road
      - Roof
    - Dynamic
      - Scissors
      - Arms on a clock
  - Qualitative comparison



# Some characteristics of slope

- ▶ Ratio
    - Proportionality
      - Multiplicative rather than additive relationship
  - ▶ Rate-of-change
    - Change of quantity over time
    - Constant quantity has rate of change equal to 0
  - ▶ Measure of steepness
    - Important graphical features of the incline
    - Determined by combination of 2 distances, not just one
    - Constant throughout the incline
- 

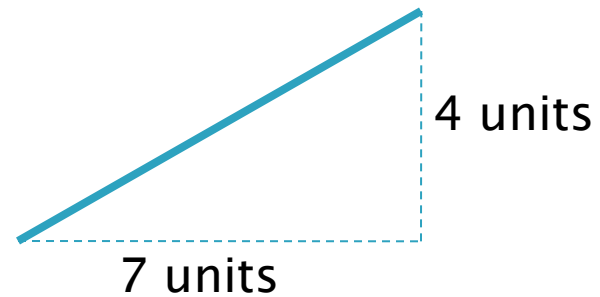
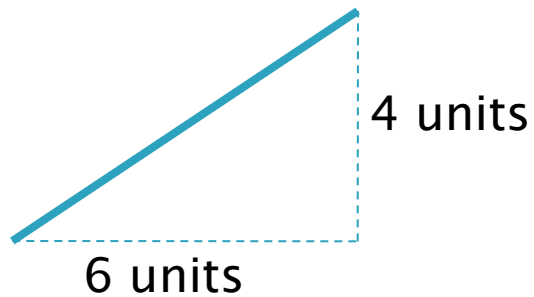
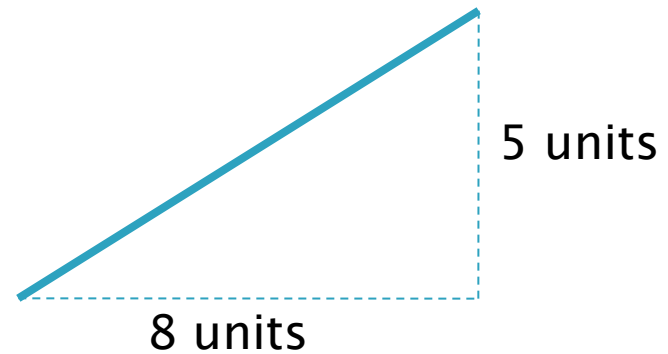
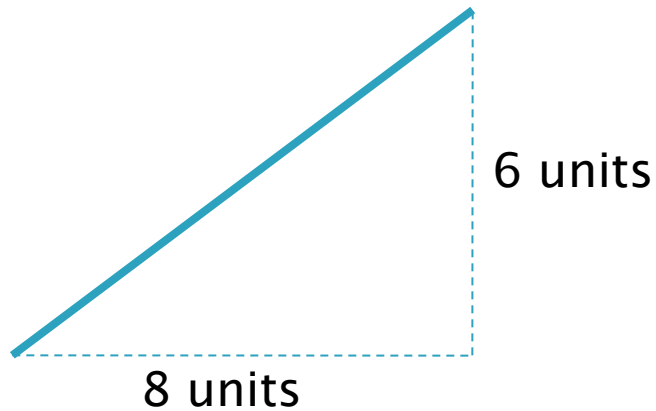
# How can we get from angle to slope?

- ▶ Steepness activities in several contexts
  - Provide problem-solving opportunities for children to analyze their understanding of steepness
- ▶ Increase structural complexity
  - How many dimensions remain constant?
  - Are the numbers nice?
    - integral ratios

# Classes of activities

- ▶ “Observe” – Comparison
  - ▶ “Construct” – Missing Value
  - ▶ “Imagine” – Visualization tasks
- 

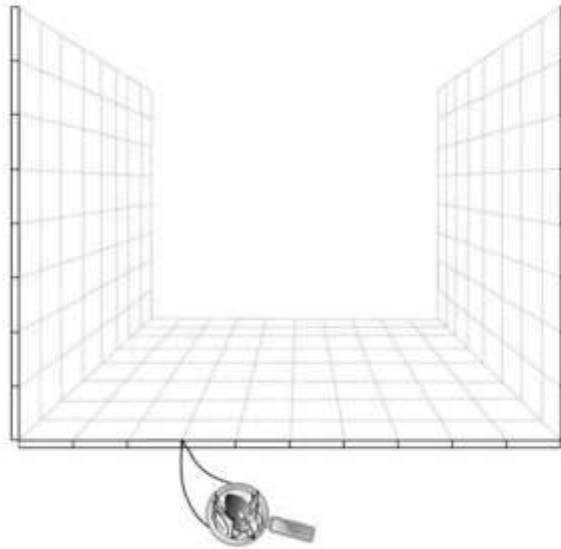
# Holding one dimension constant



# Introducing Ari & Nid

Ari  and Nid  are spiders.

Ari and Nid live in identical rooms which have the same large square tiles on the walls and on the floor.



Ants  and flies  also live in Ari and Nid's rooms.

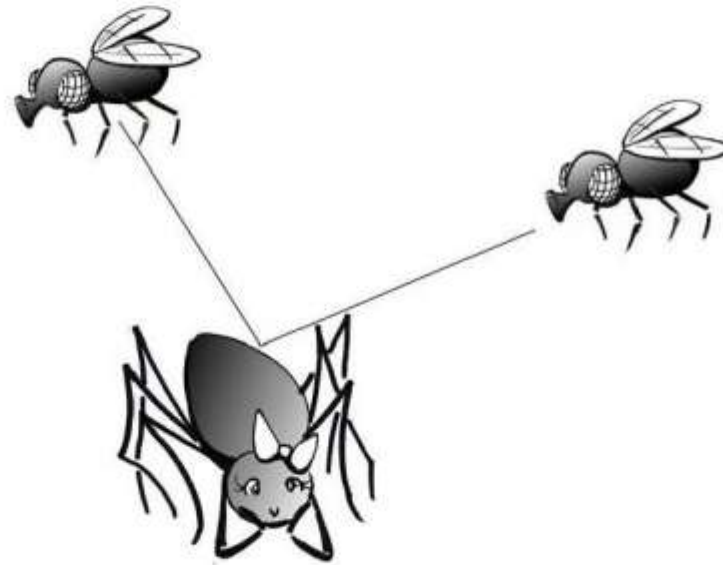
They like to hide in cracks between the tiles.

# Introducing Ari & Nid

Ari and Nid can shoot webs to catch tasty bugs.



They can shoot their webs in a straight line as far as they want in any direction. Steeper webs are more accurate.

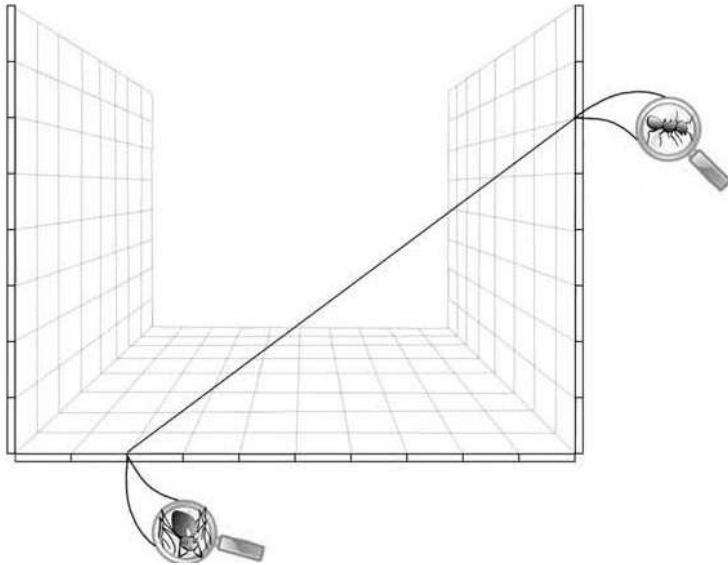


In the next activities we will ask you to help Ari and Nid decide whose web is steeper.

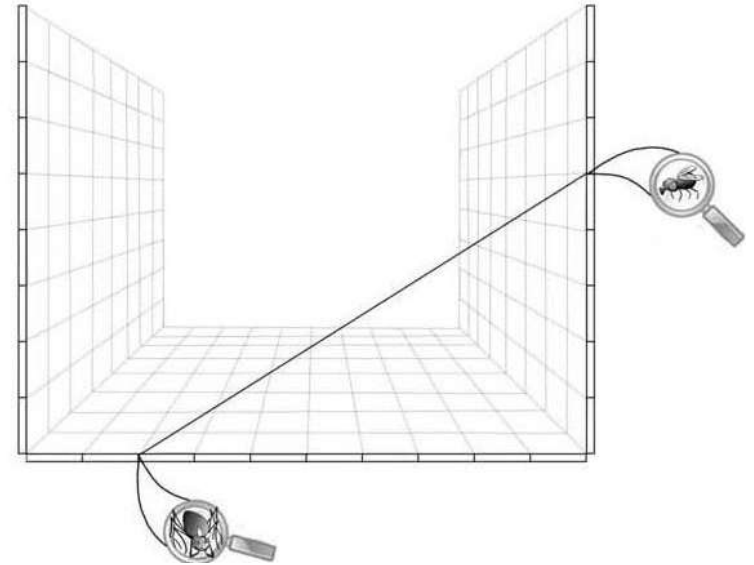
# Steepness questions in context: Ari & Nid

What can you say about the steepness of the spiders' webs?

Ari's Web



Nid's Web



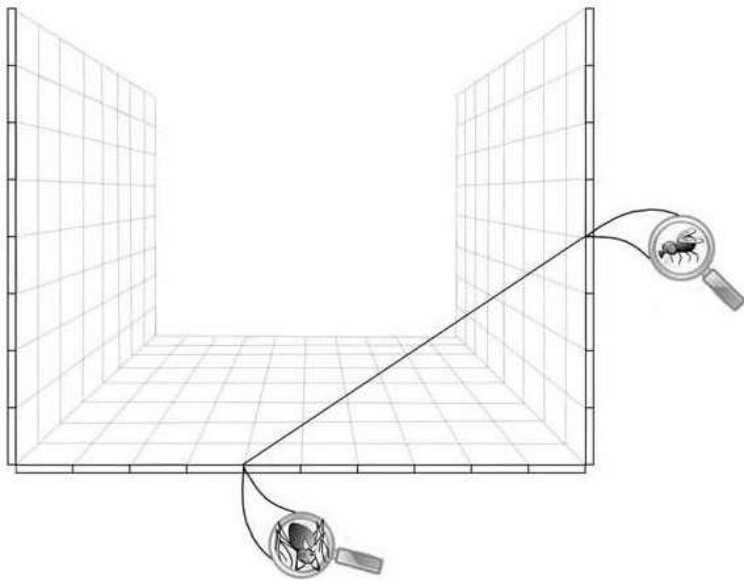
Choose the best answer:

- A. Ari's web is steeper.
- B. Nid's web is steeper.
- C. The webs are the same steepness.
- D. It is not possible to tell which web is steeper.

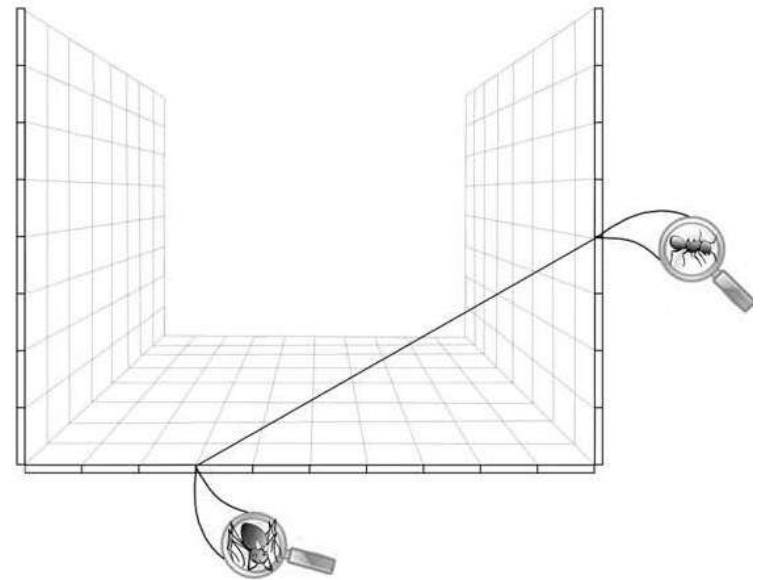
# Steepness questions in context: Ari & Nid

What can you say about the steepness of the spiders' webs?

Ari's Web



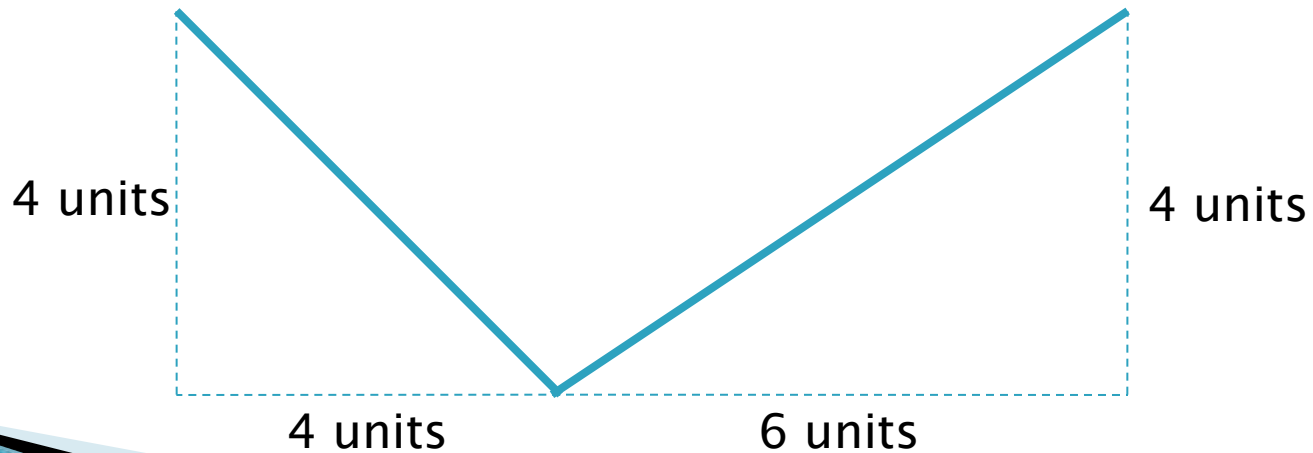
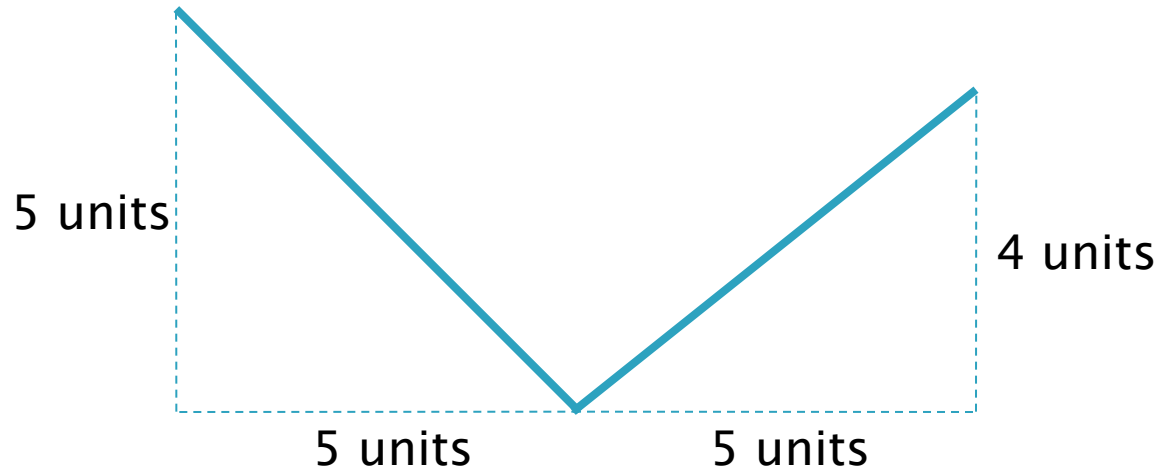
Nid's Web



Choose the best answer:

- A. Ari's web is steeper.
- B. Nid's web is steeper.
- C. The webs are the same steepness.
- D. It is not possible to tell which web is steeper.

# Direction vs. Steepness

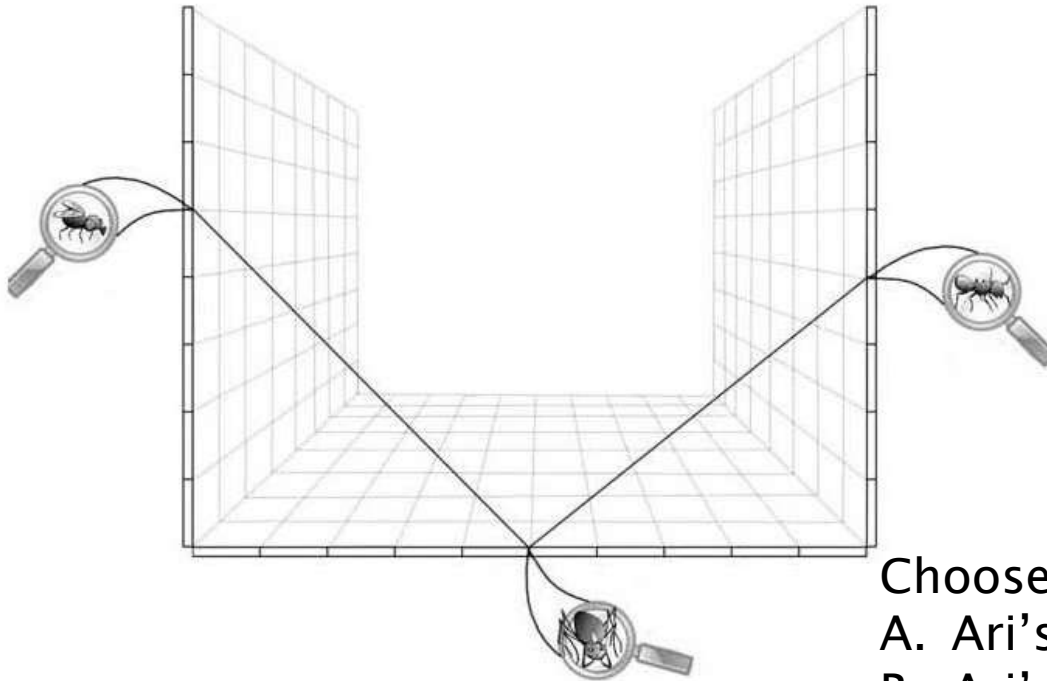


# Steepness questions in context: Ari & Nid

What can you say about the steepness of Ari's webs?

LEFT Web

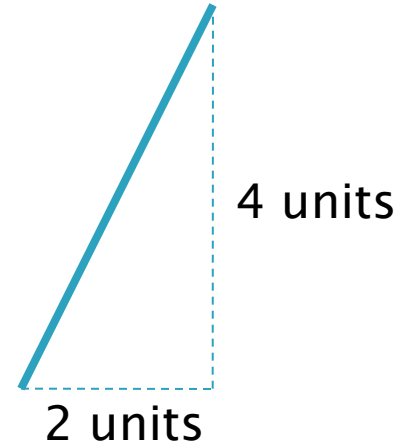
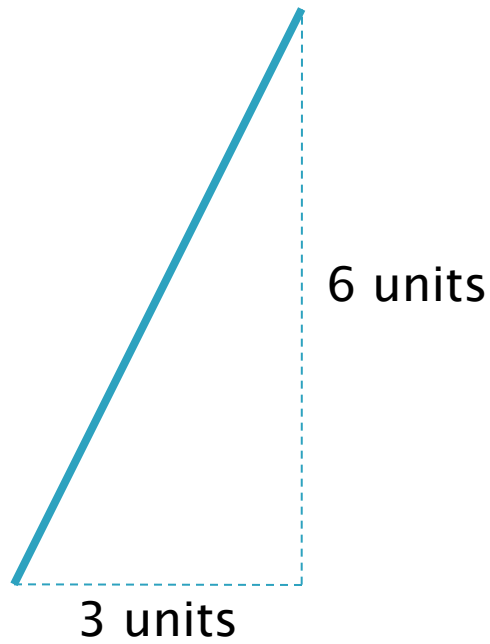
RIGHT Web



Choose the best answer:

- A. Ari's LEFT web is steeper.
- B. Ari's RIGHT web is steeper.
- C. The webs are the same steepness.
- D. It is not possible to tell which web is steeper.

# Similar Triangles




# Introducing Natasha's and Shivi's Roofs

Natasha and Shivi like to build toy houses with toy bricks. They have lots of bricks of the same size.

When they finish building, Natasha and Shivi compare their houses.

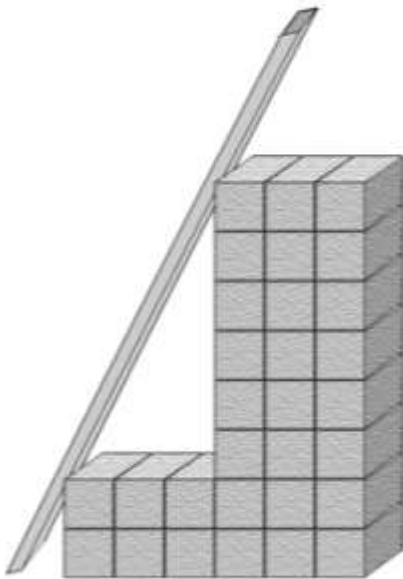
Natasha and Shivi heard that steeper roofs are safer during a snow storm. They would like to check whose roof is steeper.



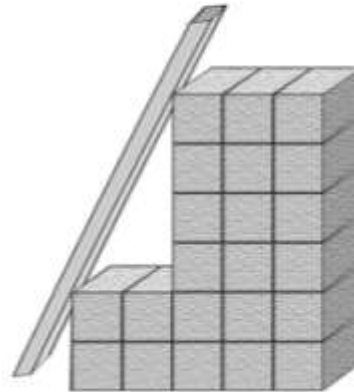
# Steepness questions in context: Roofs

What can you say about the steepness of Natasha's roof and Shivi's roof?

NATASHA'S ROOF



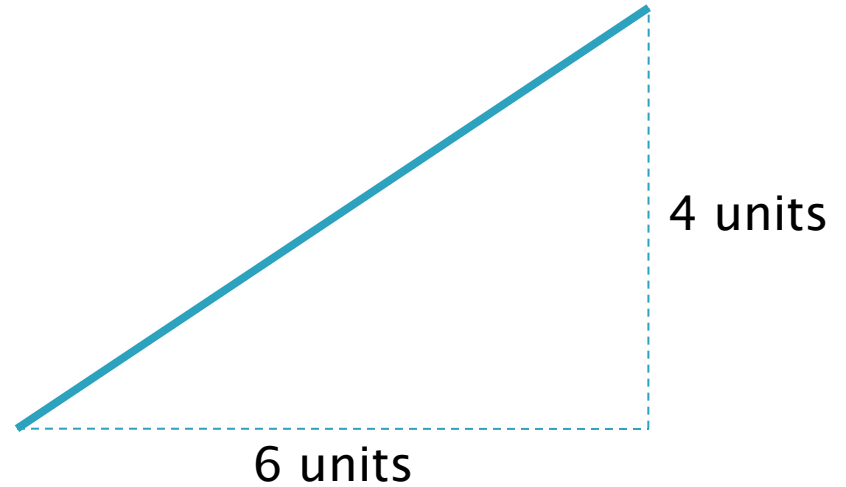
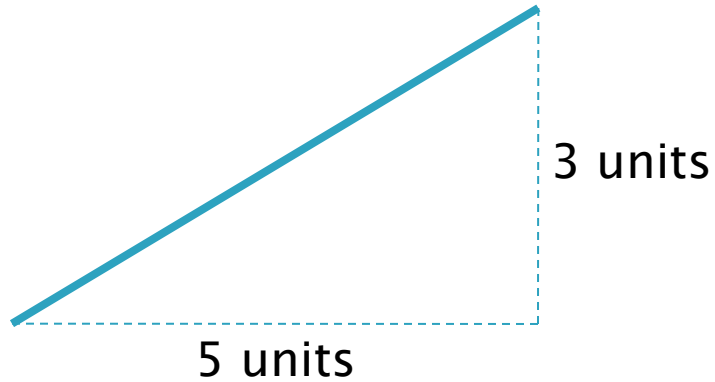
SHIVI'S ROOF



Choose the best answer:

- A. Natasha's roof is steeper.
- B. Shivi's roof is steeper.
- C. The roofs are the same steepness.
- D. It is not possible to tell which roof is steeper.

# Non-similar triangles & no constant sides




# Introduce Dulani's and Mei-Li's Ramps

Dulani and Mei-Li like to make ramps. They have lots of identical cube blocks that they use to build towers for their ramps.

Mei-Li and Dulani like to check whose ramp is steeper.

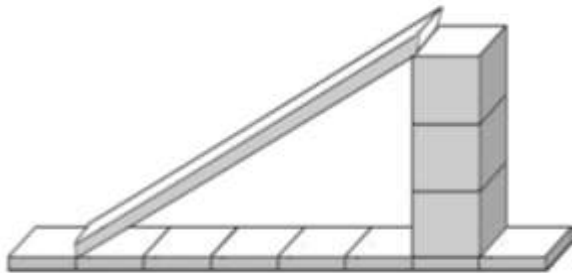
In the next activities we will ask you to help Mei-Li and Dulani decide whose ramp is steeper.



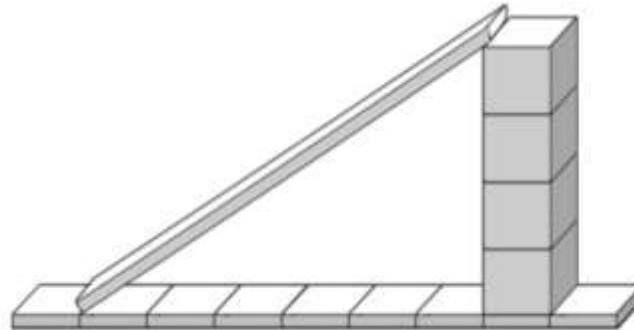
# Steepness questions in context: Ramps

What can you say about the steepness of Mei-Li and Dulani's ramps?

Mei-Li's ramp



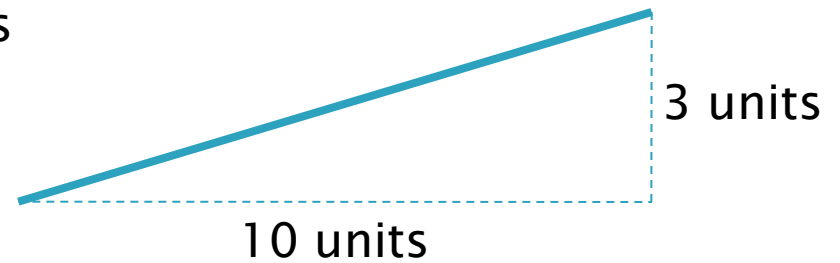
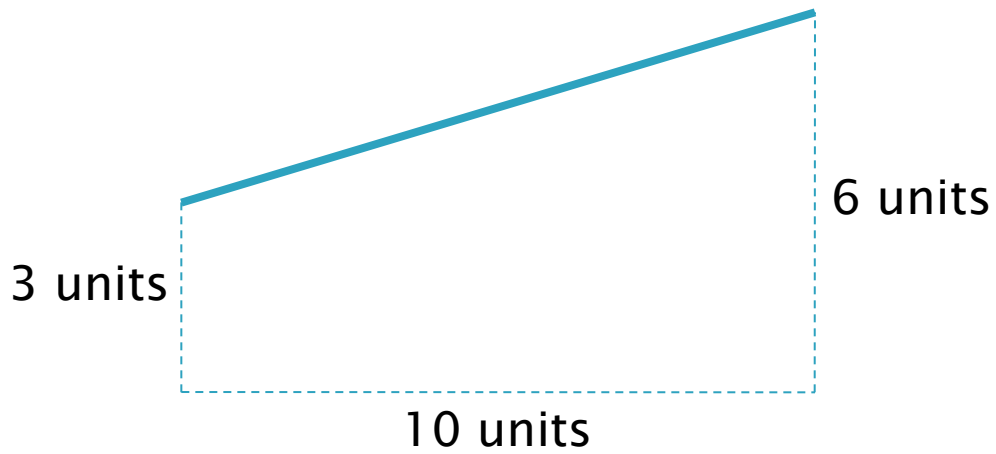
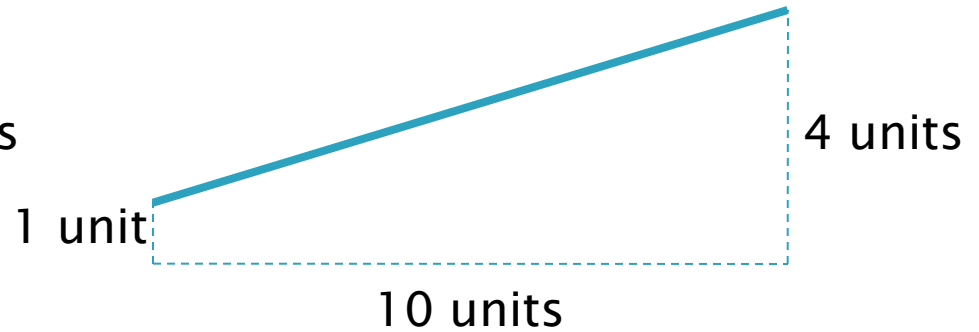
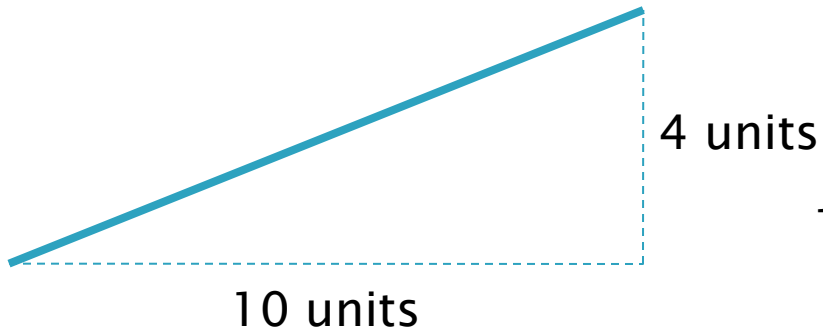
Dulani's ramp



Choose the best answer:

- A. Mei-Li's ramp is steeper.
- B. Dulani's ramp is steeper.
- C. The ramps are the same steepness.
- D. It is not possible to tell which ramp is steeper.

# Non-zero intercept



# Introducing String Patterns

Beatriz likes to make string patterns for her friends.

She needs your help in making some string patterns.

Beatriz uses many different colors of string. Today she wants to use **blue** string when the string is steeper and **red** string when it is less steep.

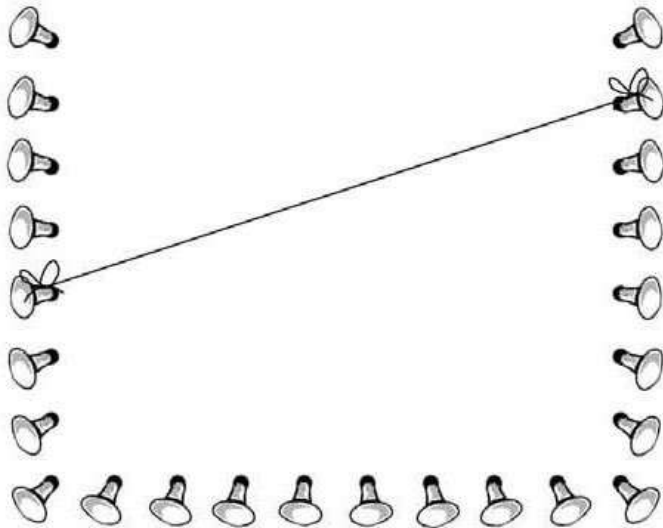
In the next activities we will ask you to help Beatriz decide which string is steeper.



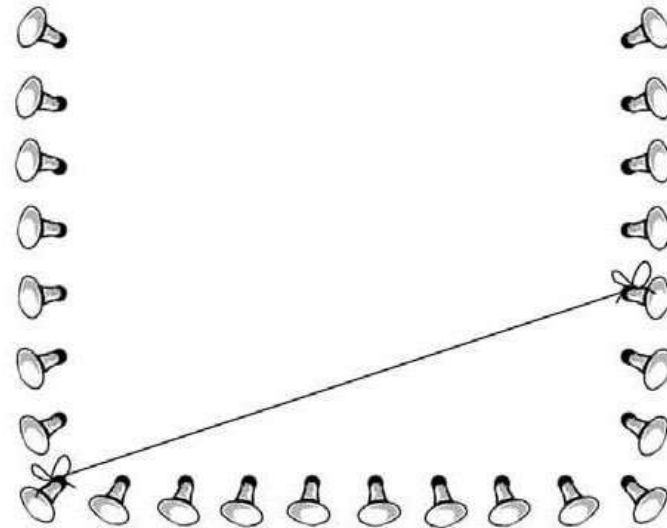
# Steepness questions in context: String Patterns

What can you say about the steepness of the two strings?

LEFT Pattern



RIGHT Pattern



Choose the best answer:

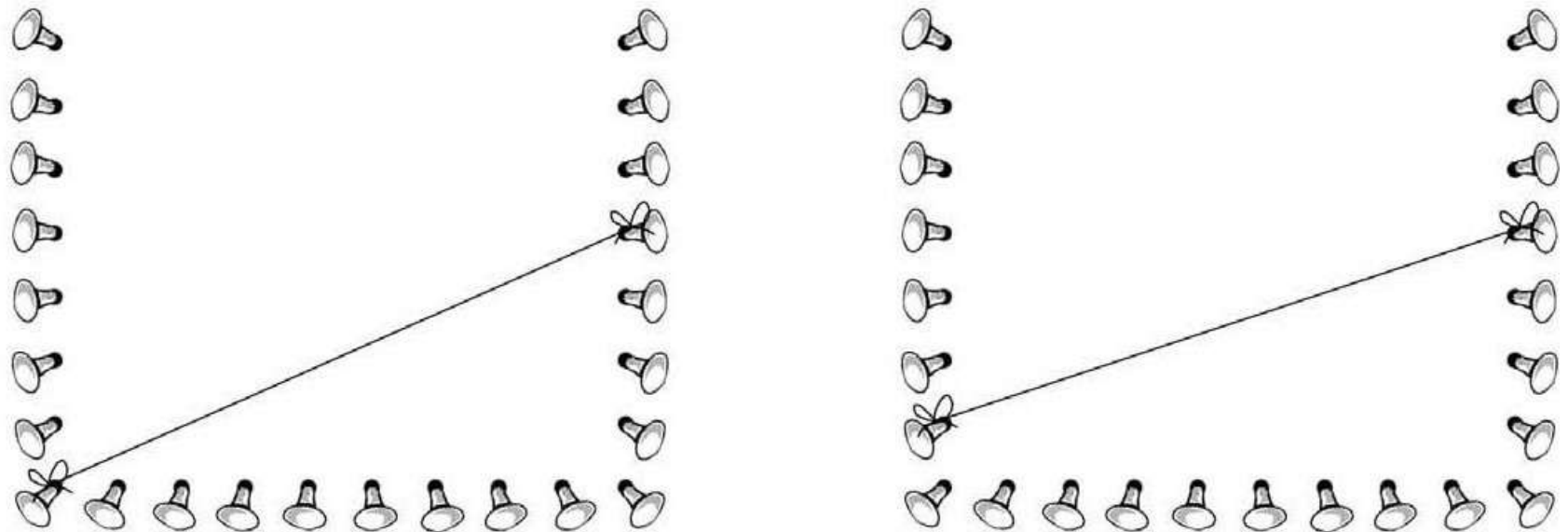
- A. The string in the LEFT pattern is steeper.
- B. The string in the RIGHT pattern is steeper.
- C. The strings are the same steepness.
- D. It is not possible to tell which string is steeper.

# Steepness questions in context: String Patterns

What can you say about the steepness of the two strings?

LEFT Pattern

RIGHT Pattern



Choose the best answer:

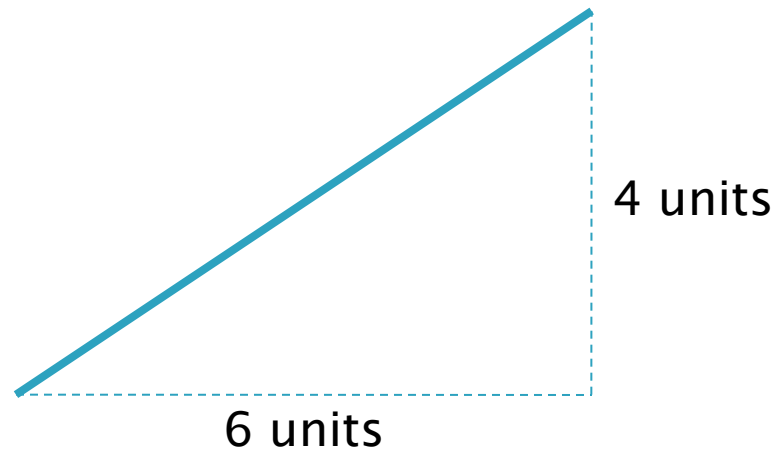
- A. The string in the LEFT pattern is steeper.
- B. The string in the RIGHT pattern is steeper.
- C. The strings are the same steepness.
- D. It is not possible to tell which string is steeper.

# Hands-on Ramp Interviews



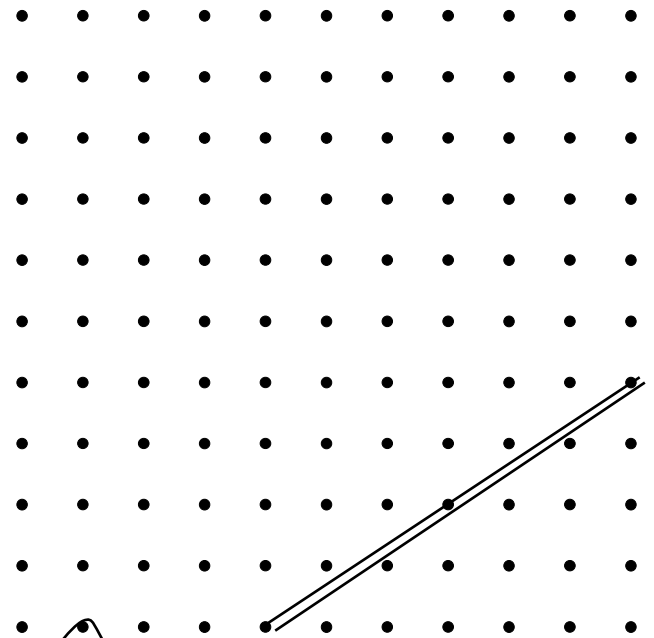
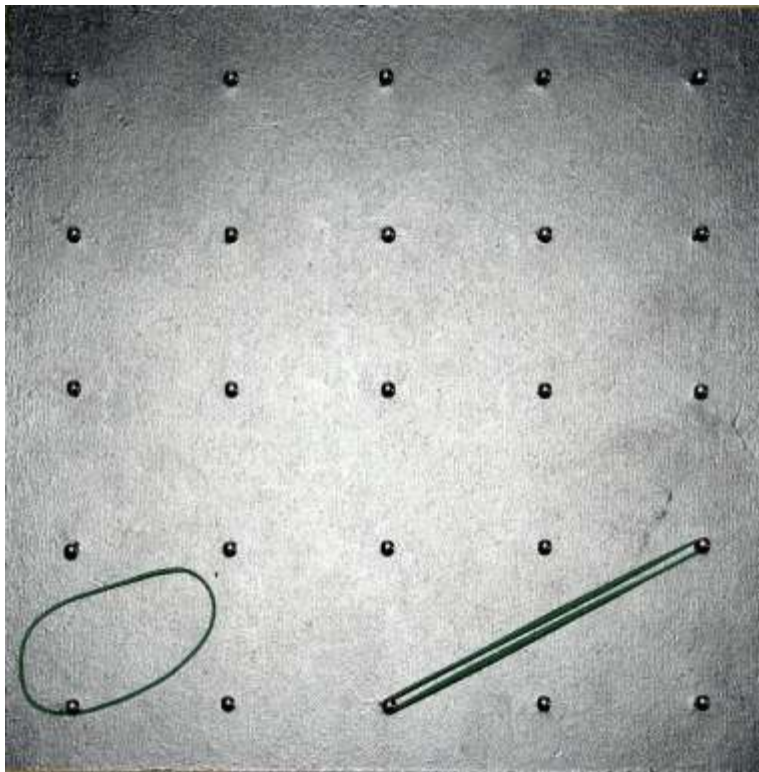
# Construct – Missing Value

- ▶ Draw a line which is the same steepness as shown below but measures 9 units on the horizontal.



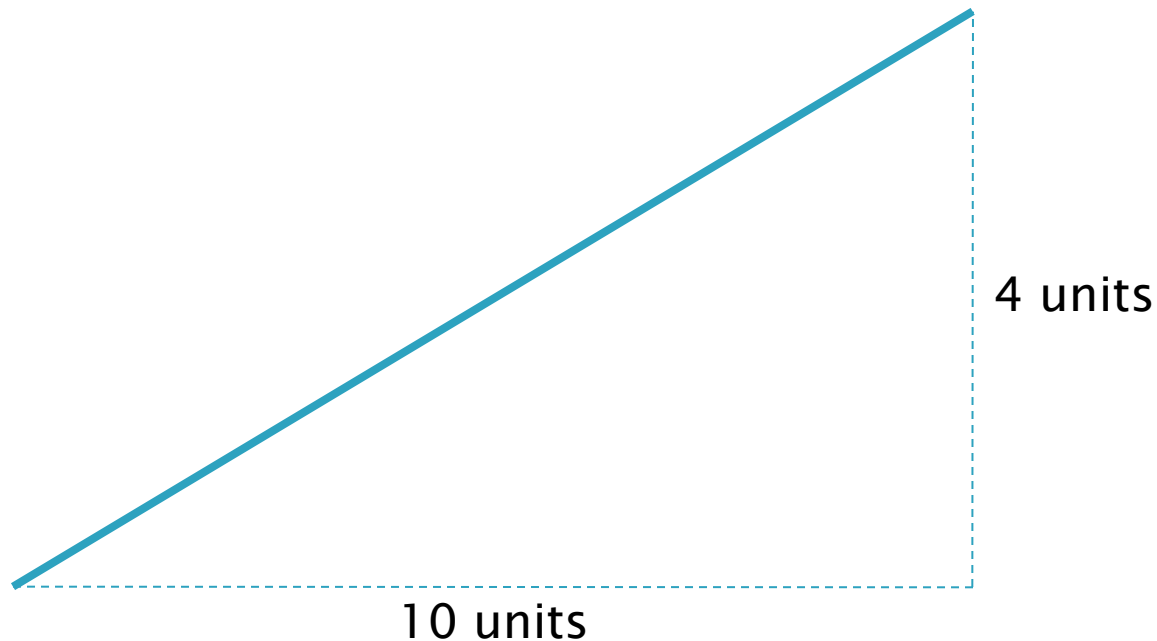
# Steepness questions in context: Geoboards

Make one line that is the same steepness as the line shown below, but starts at the nail where the second elastic band is attached.



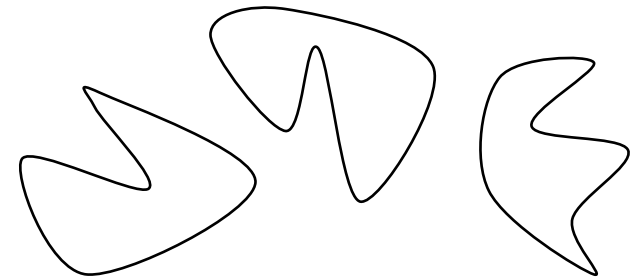
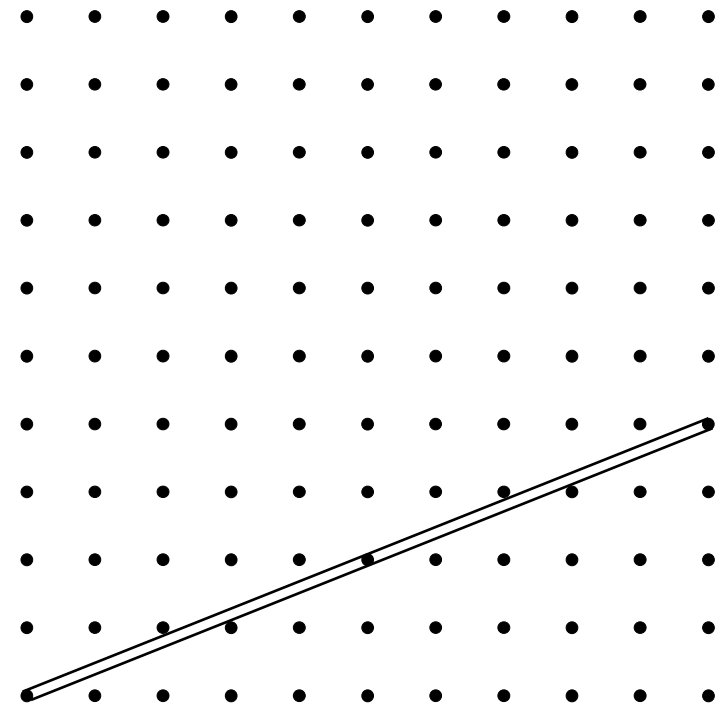
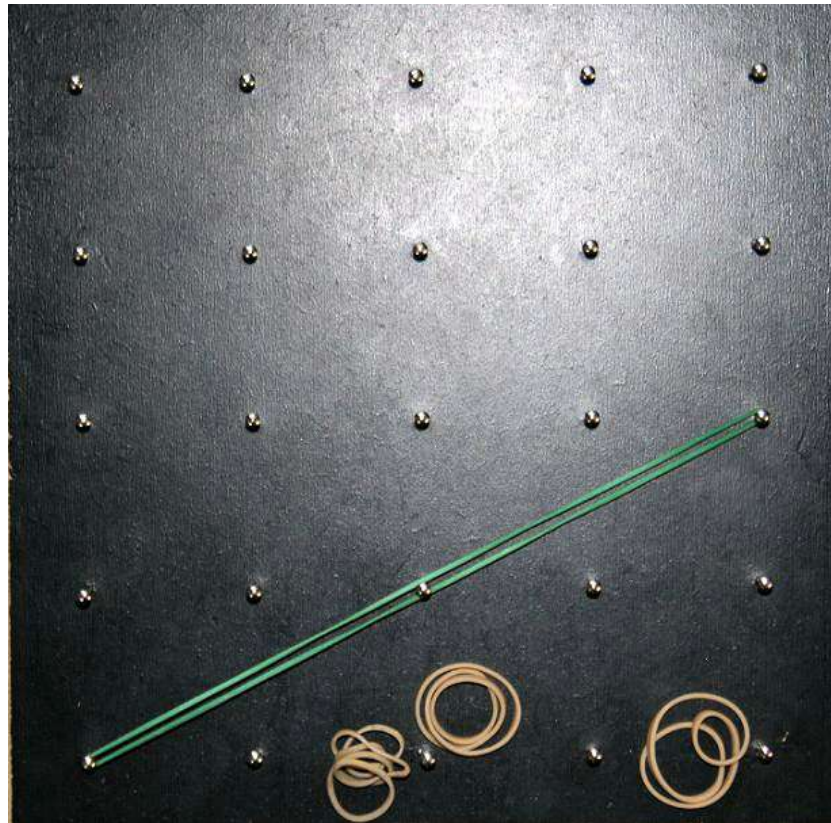
# Construct – Open ended

- ▶ Draw 3 lines which are the same steepness as shown below.



# Steepness questions in context: Geoboards

Make 3 lines that are the same steepness as the line shown below.



# Imagine – Visualization

## ▶ Sample questions

### ◦ Videotape Ramps

- I made two (2) ramps using the red and the yellow boards (same length). For the red ramp I used 12 videos to build the tower. For the yellow ramp I used 13 videos. Can you tell which ramp is steeper?
- I took those ramps apart and made two (2) new ramps. One board I used was 20 inches long. The other board I used was 10 inches long. Can you tell which ramp is steeper?

# Imagine – Visualization

## ▶ Sample questions

### ◦ String context

- Beatriz wants to make two strings from the same place on the bottom of her pattern. She wants the green string to go to 4 nails up on the side and the orange string to go to 5 nails up on the side. Which string will be steeper?
- Beatriz finds an old pattern that she made. In it, the black string goes to 6 nails up on the side. The purple string goes to 2 nails up on the side. Which string is steeper?

# Misconceptions of steepness

## ▶ Area

- Area 'under' the incline is proportional to steepness

## ▶ Height

- The height of the incline is proportional to steepness

## ▶ Speed

- Objects rolls faster down steeper ramps

## ▶ Additive

- To preserve steepness, add the same number of units to the horizontal and vertical distances

# Working with steepness tasks

- ▶ Observed misconceptions
  - In individual paper-and-pencil activity in grades 5–9
  - In one-on-one interviews using the videotape ramps in grades 2 – 7
- ▶ Corrected misconceptions
  - Group interview activities bases around Ari & Nid questions in grades 5 and 8

# More information

- ▶ Activities in 4 contexts
  - ▶ Mathematics behind the contexts
    - ▶ PowerPoint presentation
- are available at:  
[alum.mit.edu/www/dianasc](http://alum.mit.edu/www/dianasc)

Diana Cheng

[DianaSC@bu.edu](mailto:DianaSC@bu.edu)

Polina Sabinin

[Polina@Sabinin.info](mailto:Polina@Sabinin.info)